(c_{ij} being the elements of the transfer matrix of a two-arm junction) and

$$P = dd_r.$$

We have pointed out that to have D_1 and D_2 belonging to the same family it is necessary and sufficient that the corresponding matrices P verify

$$P_2 = q_1 P_1 q_1^{-1}.$$

Therefore, all the elements of the family have the same equation:

$$\det. (\lambda I - P) = \lambda^2 - [|c_{11}|^2 + |c_{22}|^2 - |c_{12}|^2 - |c_{21}|^2]\lambda + 1 = 0,$$

and the expression

$$T = |c_{11}|^2 + |c_{22}|^2 - |c_{12}|^2 - |c_{21}|^2$$

is an invariant in the family.

1) $T>2$ is the case that Beatty[1] has considered. By the diagonalization of P, the intrinsic losses are easily set up.
2) $T=2$ cannot be diagonalized. One can decompose the two-arm junction into a complex admittance preceded and followed by nondissipative two-arm junctions (one of them being a length of lossless waveguide), and if this two-arm junction is symmetric, it can be decomposed into a complex admittance preceded and followed by two equal lengths of lossless waveguide. In this case, we can theoretically reduce the insertion loss to a value as small as we want.

These notions will be generalized to the m-arm junctions in a later paper.

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Correction to "Superheterodyne Radiometers for Use at 70 Gc and 140 Gc"[1]

In Appendix II of the above paper, (9) and (10) should read

$$\omega_s = 2\omega_c + \omega_{\mathrm{IF}} \tag{9}$$

$$\omega_i = 2\omega_c - \omega_{\mathrm{IF}}. \tag{10}$$

Each n in (15) and (17) should be an η; and, in the equation immediately above (18), the large curly bracket should be an integral sign.

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Manuscript received March 23, 1964.

[1] R. Meredith and F. L. Warner, "Superheterodyne radiometers for use at 70 Gc and 140 Gc," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 397–411; September, 1963.

Measurements of Varactor Diode Impedance

Summary

Impedance measurements of varactor diodes have been made on a slotted line in the frequency range 2–18 Gc. The experimental technique is described, and a method for deriving a simple lumped equivalent circuit is shown. Typical circuit values are quoted.

Introduction

In earlier microwave measurements of varactor diodes, *e.g.*, by Houlding,[1] Harrison,[2] and Mavaddat,[3] the work has been done in such a way that the circuit values derived have been those of the actual semiconductor junction, rather than over-all values for the complete encapsulated diode. In general, the method used involved matching the varactor diode and its mount to the waveguide or line and studying the variations in impedance at a chosen reference plane as the bias was altered. The measurements have usually been made at a single frequency.

This approach to the problem has difficulties; there are, for example, uncertainties due to losses in matching components and in the correct choice of reference planes. It is nevertheless quite helpful to varactor diode manufacturers as the technique gives results which apply to the semiconductor junction alone, regardless of the encapsulation.

The microwave circuit designer has, however, almost the opposite problem. He is particularly concerned with the impedance properties of the encapsulated varactor package for various ranges of frequencies. The method described in this communication was developed with this purpose in mind and the results should be of particular interest to the designers of parametric amplifiers.

A direct microwave measurement technique was employed, using normal methods for measuring high mismatch on a slotted line. The results showed that the varactors could be successfully represented by a simple lumped constant circuit, with circuit values close to those quoted by the manufacturers for the junction.

Measurement Technique

The physical form of the diodes measured is as shown in Fig. 1(a). In early experiments, the diode was mounted in a simple collet, and measured on a 50-ohm slotted line, with an N-type connector between the diode and the line. The results of these experiments showed that the connector and the collet caused an appreciable perturbation of the results, and a simpler mounting arrangement was devised. This is shown in Fig. 1(b). The diode was mounted directly at the end of the 50-ohm slotted line, being held in position by gentle pressure from the inner conductor. In this arrangement the diameter of the inner conductor of the line is the same, or very nearly the same, as the diameter of the end caps of the diodes. This avoids discontinuity capacitances which might be confused with the stray capacity of the diode itself. A reference plane was chosen 0.020 inch inside the diode as shown in Fig. 1(b), and a short circuit could be provided across the line at this plane, which corresponds to the inner edge of the brass cap. When this short circuit was in place, the position of this reference plane, with respect to the scale of the slotted line, could be found by measuring positions of minima.

Having established the reference plane, impedance measurements of the diodes could be made by normal slotted line methods. However, several precautions had to be taken.

First, the input power had to be kept below a certain level, as the impedance of the varactor diode was affected by power level. This upper limit of power was that, giving a voltage of 1 mv across the diode. A convenient way of ensuring that the power was low enough was to use the slotted line method in which source and detector of the slotted line are interchanged.[1] In this method, the load and the detector operate at comparable power levels; in some other methods the power level at the probe is some 20 or 30 db below that at the load and it is more difficult to obtain sufficient sensitivity.

Second, the VSWR of the diodes was very high. Because of this, the double minimum method[4] of impedance measurement was used. In this method, measurement is made of the distance Δx between the two symmetrical carriage positions about a minimum for which the detector reading is twice the minimum reading. For a square-law detector, provided $S>10$

$$\text{VSWR} \qquad S = \frac{\lambda}{\pi \Delta x}$$

where λ is the wavelength. This method is better for large VSWR values than the conventional one, which would demand calibration of the detector system over a wide dynamic range.

When this method is used with source and detector in the normal positions, the errors due to probe coupling are very low, as the probe is operated in a region of low impedance. Similarly with the probe-feed arrangement, tight probe coupling will introduce little error, as the impedance seen by the probe in the region of the minimum differs little from the probe capacitance only. This gives the constant-current source that this method requires to be accurate. Fig. 2 shows the experimental arrangement. An unmodulated source was used, and the detector output was measured on a valve voltmeter (V.V.M).

Results

Fig. 3, page 470, shows some typical results obtained with diode "A." The line is drawn dashed beyond the 12-Gc point since

Manuscript received September 13, 1963; revised March 23, 1964.

[1] N. Houlding, "Measurement of varactor quality," *Microwave J.*, vol. 3, p. 40; 1960.

[2] R. I. Harrison, "Parametric diode Q measurements," *Microwave J.*, vol. 3, p. 43; 1960.

[3] R. Mavaddat, "Diode Q-Factor Measurements," University College of North Wales School of Applied Science; February, 1963.

[4] E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Company, Inc., New York, N. Y., p. 266; 1957.

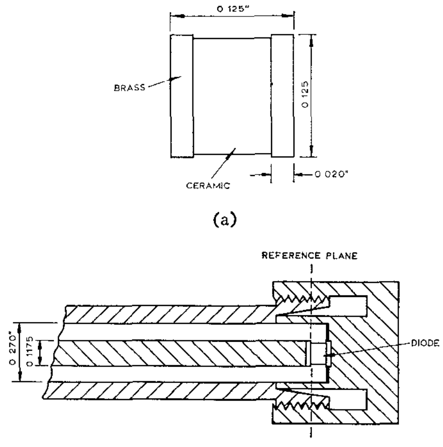


Fig. 1—(a) Physical form of varactor diode. (b) Mounting of diode at end of slotted line.

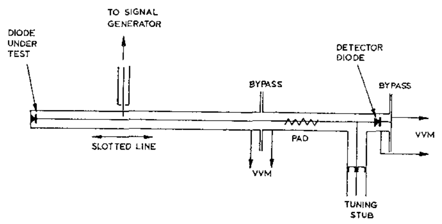


Fig. 2—Experimental arrangement.

the particular slotted line suffered from spurious mode operation above this frequency. By plotting the angle on the chart against frequency, it was possible to obtain a good estimate of the higher frequency at which the impedance is resistive. Assuming that the diode has the simple equivlaent circuit shown in Fig. 5(a) and following the method of calculation given in the Appendix, then from (5), (6) and (7)

$$f_1 = 6 \text{ Gc}$$
$$f_2 = 15 \text{ Gc}$$
$$X_0 = -38.7\ \Omega$$
$$S_0 = 24$$

whence

$$C_1 = 0.17 \text{ pf}$$
$$C = 0.90 \text{ pf}$$
$$L = 0.79 \text{ nh}$$
$$R = 2.1\ \Omega.$$

Results obtained with another diode ("B") are also shown as a Smith Chart plot in Fig. 4. Before these results were taken, the slotted line had been sleeved and given a new inner conductor so that the principal mode only would propagate at frequencies up to 18 Gc.

The values calculated for this diode from Fig. 4 are

$$C_1 = 0.13 \text{ pf}$$
$$C = 0.73 \text{ pf}$$
$$L = 0.79 \text{ nh}$$
$$R = 1.7\ \Omega.$$

It is interesting to note that the values for C are within 10 per cent of those obtained from bridge measurements at 800 Mc.

The crosses on Figs. 3 and 4 show theoretical impedances calculated from the derived circuit values. It can be seen that this simple equivalent circuit does in fact give a very fair description of the behavior of the diode impedance with frequency. One criticism that may be made of the validity of the simple equivalent circuit is that it suggests a higher VSWR in the region of the high-frequency resonance than is measured in practice. For example, the values shown above for diode A would correspond to a VSWR of 50 at f_2, whereas the measured value was 15.

This discrepancy at the higher frequencies could be accounted for by a shunt loss resistance of the order of 1000 Ω, assuming the measuring line to have no losses. In fact there was a small loss in the line giving VSWR values between 50 and 100 for true open and short circuits.

Conclusions

Basic microwave slotted line methods have been extended and adapted to measuring varactor diodes for a range of frequencies between 2 and 18 Gc. The diode is measured between the terminals by which it is connected to the microwave circuit. This gives the circuit designer the information he needs in order to suit his circuit to the diode.

A simple equivalent circuit can be found which gives a fairly good description of the behavior of the diode impedance with frequency, particularly if shunt loss is added at high frequencies.

Appendix

Assume that the varactor diode has the equivalent circuit shown in Fig. 5(a), page 472. Then

$$Y = j\omega C_1 + \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$
$$= j\omega C_1 + \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

The admittance will be a pure conductance when

$$C_1 + \frac{(1 - \omega^2 LC)C}{(\omega CR)^2 + (1 + \omega^2 LC)^2} = 0,$$

i.e., when

$$2\omega^2 = \frac{2}{LC} + \frac{1}{LC_1} - \left(\frac{R}{L}\right)^2 \pm \sqrt{\left(\frac{2}{LC} + \frac{1}{LC_1} - \frac{R^2}{L^2}\right)^2 - \frac{4(C_1 + C)}{(LC)^2 C_1}}.$$

If R is small enough, $(R/L)^2$ can be neglected in comparison with the other terms. (This is normally so for varactor diodes.) The expression then becomes

$$2\omega^2 = \frac{2}{LC} + \frac{1}{LC_1} \pm \frac{1}{LC_1},$$

so that the two resonant frequencies are given by

$$\omega_1^2 = \frac{1}{LC} \tag{1}$$

$$\omega_2^2 = \frac{1}{L}\left(\frac{1}{C} + \frac{1}{C_1}\right), \tag{2}$$

as can indeed be seen by inspection of the circuit where R is taken as zero.

From the measurement of ω_1 and ω_2 and the reactance at some known frequency it is possible to determine C, C_1 and L. Let us choose the frequency

$$\omega_0 = \frac{\omega_1}{2} = \frac{1}{2\sqrt{LC}}.$$

Then

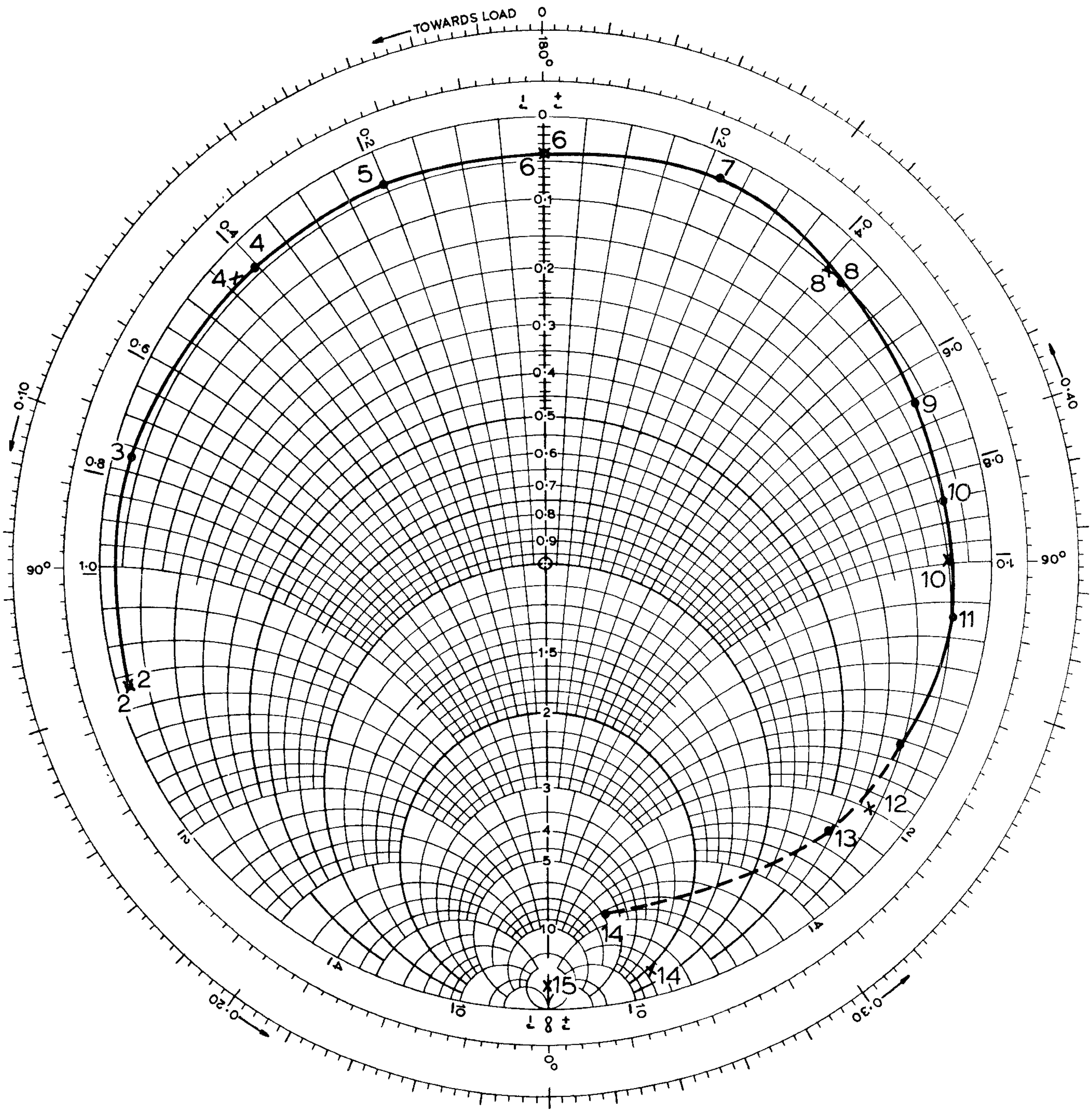
$$Y_0 = j\frac{\omega_1}{2} C_1 + \frac{R - j\left(\frac{\omega_1}{2} L - \frac{2}{\omega_1 C}\right)}{R^2 + \left(\frac{\omega_1}{2} L - \frac{2}{\omega_1 C}\right)^2}$$
$$= \frac{jC_1}{2\sqrt{LC}} + \frac{4CR + j6\sqrt{LC}}{9L + 4CR^2}.$$

Expressing Y_0 as $G_0 + jB_0$ and neglecting the term in R^2, as R is small

$$G_0 = \frac{4CR}{9L}$$
$$B_0 = \frac{1}{\sqrt{LC}}\left(\frac{C_1}{2} + \frac{2C}{3}\right)$$
$$= \omega_1\left(\frac{C_1}{2} + \frac{2C}{3}\right), \tag{3}$$

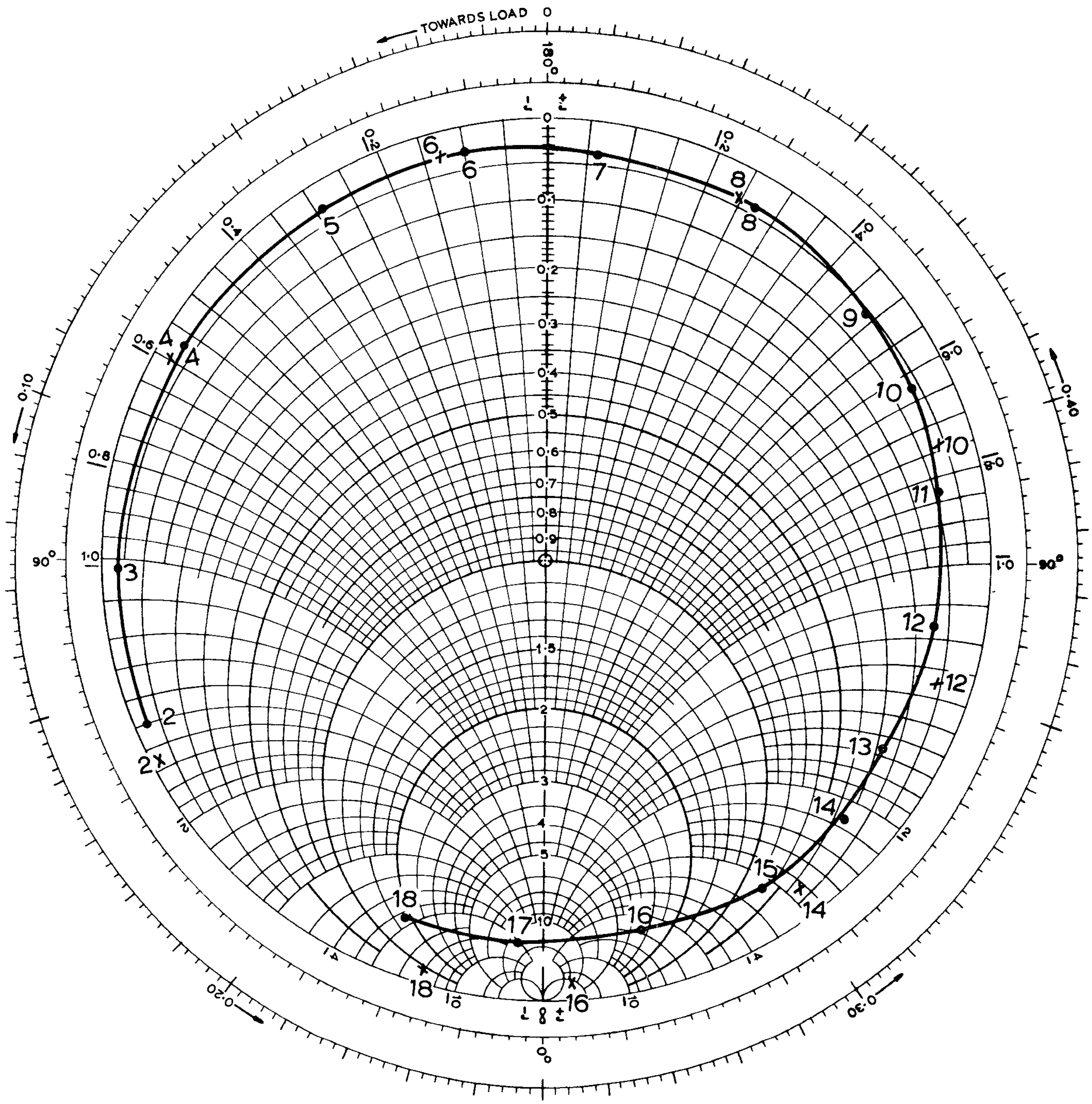
and from (1) and (2)

$$\frac{C}{C_1} = \left(\frac{\omega_2}{\omega_1}\right)^2 - 1. \tag{4}$$



Frequencies are shown in Gc
• Measured
× Calculated

Fig. 3—Impedance of diode A on 50-Ω line.



Frequencies are shown in Gc
• Measured
× Calculated

Fig. 4—Impedance of diode B on 50-Ω line.

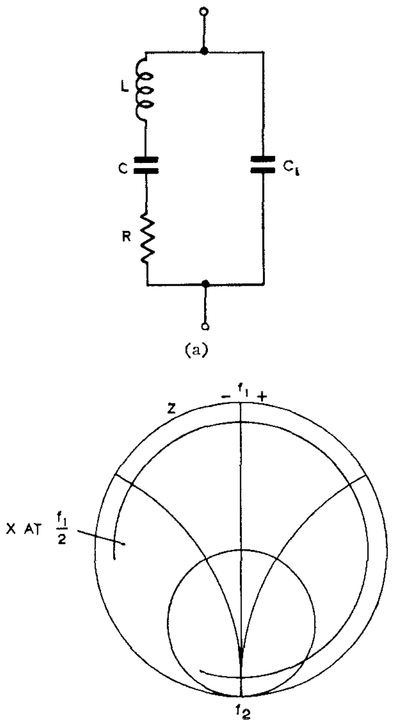


Fig. 5—(a) Equivalent circuit for varactor diode. (b) Schematic representation of impedance plot.

Now from (3) and (4)

$$C_1 = \frac{3B_0}{\pi f_1\left[4\left(\frac{f_2}{f_1}\right)^2 - 1\right]}, \qquad (5a)$$

or to a very good approximation as Y_0 is in practice almost entirely reactive,

$$C_1 = \frac{-3}{\pi f_1 X_0\left[4\left(\frac{f_2}{f_1}\right)^2 - 1\right]}, \qquad (5b)$$

where X_0 is the reactance at $f_0 = f_1/2$. Having determined C_1, (4) and (1) give

$$C = \left[\left(\frac{f_2}{f_1}\right)^2 - 1\right] C_1 \qquad (6)$$

$$L = \frac{1}{(2\pi f_1)^2 C}. \qquad (7)$$

Also, R may be determined from a direct measurement of VSWR at frequency f_1. If this is S_1,

$$R = \frac{Z_0}{S_1},$$

where Z_0 = characteristic impedance of the line.

Fig. 5(b) shows a schematic picture of the kind of Smith chart plot that would be expected from this type of equivalent circuit. f_1 and f_2 are the frequencies at which the resistive axis is crossed and $f_0 = f_1/2$.

Acknowledgment

The author wishes to thank P. A. Sowry who carefully carried out many of the measurements.

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Microwave Insertion Loss Test Set

Summary

A simple, accurate test set has been devised for measuring insertion losses at microwave frequencies. It is composed almost entirely of commercially available equipment and components. The short-term jitter is about 0.0004 db peak-to-peak, and long-term drift is typically 0.0015 db/hour. Accuracy of the measurements depends on the value of the insertion loss measured and is better than ±0.001 to ±3 per cent for insertion loss measurements in the 0 to 25 db range. These accuracies include the non repeatability of connecting and disconnecting the waveguide flanges used in the system.

The upper operating frequency is presently limited to 40 Gc by the thermistor mount characteristics of the Hewlett-Packard 431B power meter, although the test set has operated at frequencies as high as 90 Gc with transitions resulting in slight performance degradation.

Introduction

Accurate measurements of the parameters of passive microwave components are required in building and testing low-noise microwave receiving systems. At the Jet Propulsion Laboratory, Pasadena, Calif. there was a need for an instrument that could measure the insertion loss of low-loss coaxial and waveguide components beyond the normal precision capability of available equipment. Measurements of the required precision had previously been achieved[1] but at the cost of extreme care and extraordinary temperature stabilization.

A simple dual channel insertion loss test set (Fig. 1) has been constructed almost entirely from commercially available equipment and components. In this set, the signal level is sampled before application to the component under evaluation and then compared to the signal level after application. Potentiometer R1 (Fig. 1) is adjusted so that the null voltmeter indicates a null both before and after inserting the component to be measured at POINT A. The difference in the potentiometer ratio (converted to db) is a measure of the insertion loss of the component. Tables have been compiled to facilitate the conversion of the ratio reading to db.

Errors due to amplitude instability of the signal source are virtually eliminated by the test set. Modulation of the signal source is not required due to the excellent stability of the Hewlett-Packard HP 431B power meter, thus eliminating possible errors from klystron double moding, etc.

Power Meter Modifications

The internal circuitry of the power meter has an output which is proportional to the square root of the applied power. In order to have a linear output with respect to power, a squaring circuit used on the output of the power meters gives straight line segments approximating square law nonlinearity. The output linearity of the power meters was greatly improved by monitoring the feedback current through one of the power meter range resistors[2] thereby bypassing the squaring circuit output.

The bridge circuit and the necessary modifications to the power meters are shown in Fig. 2. The added resistance of the helipots assures full-scale operation of the power meters, providing the highest linearity possible. The capacitors provide noise filtering. Since the feedback current varies as the square root of the applied power, the insertion loss (in decibels) measured by the potentiometer bridge is given by 20 log (ratio).

Test-Set Performance

Fig. 3 is a photograph of the 8448-Mc microwave insertion loss test set. The VSWR, looking in either direction from POINT A, is under 1.02 to obtain the necessary absolute accuracy.[1]

To give an example of typical test-set performance, a right-angle waveguide section was measured 8 consecutive times.

1) Average insertion loss was 0.0282 db.
2) Maximum difference from this average was 0.0017 db.
3) Average difference of all measurements from 0.0282 db was 0.0008 db, which includes the nonrepeatability of connecting and disconnecting waveguide flanges.

The measurement of linearity of an insertion loss test set is very difficult. The most successful technique used an S-band precision rotary vane attenuator[3] as a standard. Table I lists the rotary vane theoretical attenuation, measured attenuation and per cent difference between the two. The per cent difference between theoretical and measured attenuation (db) is shown graphically in Fig. 4. The insertion loss test-set accuracy (derived from repeatability and linearity measurements) is somewhat better than that shown in Table II.

A selection of insertion loss measurements of H-band components is listed in Table III. Each value represents an average of ten consecutive measurements. The last column in Table III is the average value of the difference of each measurement from the average insertion loss value.

A stability test was made of the test set by recording the output of the null voltmeter, a typical sample of which is shown in Fig. 5. Short-term jitter is about 0.0004 db peak-to-peak, and long-term drift is typically 0.0015 db/hr.

Manuscript received March 16, 1964. This communication presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS 7-100, sponsored by the National Aeronautics and Space Administration.

[1] C. F. Engen and R. W. Beatty, "Microwave attenuation measurements with accuracy from 0.0001 to 0.06 db over a range of 0.01 to 50 db," *J. Res. Nat. Bur. Standards*, Section C, Engineering and Instrumentation, vol. 64C; April-June, 1960.

[2] Suggested by Mr. F. Praman of Hewlett-Packard, Palo Alto, Calif. These are resistors R160-R166 shown in Fig. 5-3 of HP Instruction Manual for the HP 431B power meter.

[3] T. Otoshi, "S Rotary Vane Attenuator," "Space Programs Summary," vol. IV, California Institute of Technology, Jet Propulsion Lab., Pasadena, Rept. No. 37-25, February 29, 1964.